ÇAĞ UNIVERSITY
FACULTY OF ECONOMICS & ADMINISTRATIVE SCIENCES

MATHEMATICS FOR BUSINESS LECTURE NOTES-7

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National Income Determination

☐ Macroeconomics is concerned with the analysis of economic theory and policy at a national level.

☐ We describe how to set up simple models of the national economy which enable equilibrium levels of income to be calculated.

☐ Initially we assume that the economy is divided into two sectors: households and firms.

☐ Firms use resources such as land, capital, labour and raw materials to produce goods and services. These resources are known as **factors of production.**

☐ **National income** represents the flow of income from firms to households given as payment for these factors.

□ Income can either be used for the consumption of goods produced by firms or it can be put into savings.

□ If we are to formulize this:

$$Y = C + S \tag{1}$$

□ where Y represents national income, C and S represent consumption and savings, respectively.

□ Let us begin with analyzing the **consumption function.**

□ Assuming the relationship between C and Y is linear, then a typical consumption function is :

$$C = aY + b .$$

□ Figure 1. shows this relationship:

$$C = aY + b$$
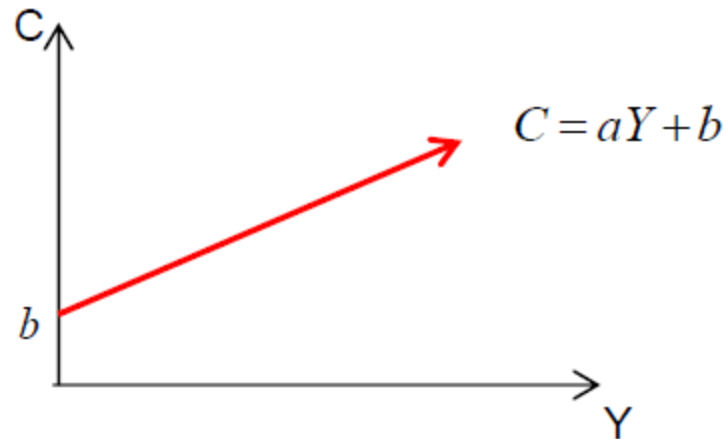
□ The intercept *b* is the level of consumption when there is no income (that is when *Y = 0*) and is known as **autonomous consumption.**

□ The slope, *a ,* is the change in *C* brought about by 1 unit increase in *Y* and is known as **marginal propensity to consume (MPC).**

☐ Please note that only a proportion of the 1 unit increase in income is consumed; the rest goes into savings.

☐ Thus, the slope, *a* , is generally smaller than 1: that is *a<1*, and definitely greater than 0, so *0<a<1.*

☐ The relation between consumption and saving *(Y= C + S)* enables the precise form of the savings function to be determined from any given consumption function.

Example 1

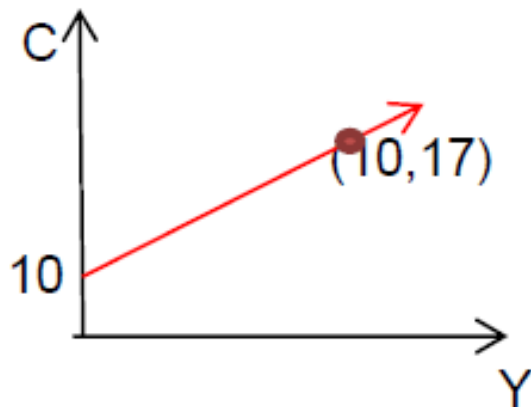a) Sketch a graph of the consumption function

$$C = 0.2Y + 15$$

b) Determine the corresponding savings function and sketch its graph.

Solution 1

□ *C = 0.2Y+15* is a linear function. In order to sketch its graph we need at least two points which pass through the line.

when $Y = 0$ then $C = 15$, we need one more point so,

when $Y = 10$ then $C = 17$

Solution 1

In order to find the savings function we use the relation:

$$Y = C + S$$

Which gives

$$S = Y - C \qquad \text{(subtract } C \text{ from both sides)}$$

$$= Y - (0.2Y + 15) \quad \text{(substitute } C \text{)}$$

$$= Y - 0.2Y - 15 \quad \text{(multiply out the bracket)}$$
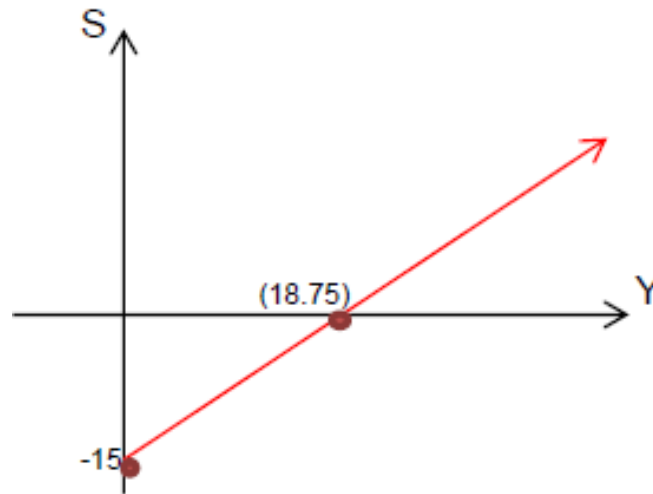
$$= 0.8Y - 15 \qquad \text{(collect like terms)}$$

We now need at least two points that pass through the line $S = 0.8Y - 15$

when $Y = 0$ then $S = -15$, we need one more point so,

when $S = 0$ then $Y = 18.75$

Solution 1

□ **Important:** One interesting feature, which contrasts with other economic functions considered so far, is that it is allowed to take negative values. This is to be expected because whenever consumption exceeds income, households must finance the excess expenditure by withdrawing savings. In other words, saving can get negative values as you can see from saving-income graph.

Practice 1

☐ Determine the savings function that corresponds to the consumption function.

a) C=0.5Y-10
b) C=0.7Y+20

Practice 2 (for students)

☐ Determine the consumption function that corresponds to the savings function

a) $S = 0.8Y - 56$

b) $S = 0.4Y + 36$

- Remember that the slope, *a* , of the consumption function is called **marginal propensity to consume (MPC).**

- Similarly, the slope of the corresponding saving function *(1- a)* is known as **marginal propensity to save (MPS).**

- The slope of the consumption function, *a* , and the slope of the corresponding saving function, *(1- a),* always add up to 1:

$$MPC + MPS = 1 \tag{2}$$

Investment, government expenditure, taxation, disposable income

- So far, we simply modeled national income with consumption and savings.

- This is crude since it fails to take into account government activity or foreign trade.

- We simply assumed that the national income can be used for either consumption or savings. That is before we take investment into account.

- The national income can also be used for consumption and investment.

$$Y = C + I \qquad (3)$$

- where *I* stands for investment.

□ When this is the case, we must then take the government expenditure and taxation into account to be more realistic:

$$Y = C + I + G \tag{4}$$

□ where *G* stands for government expenditure.

□ We must also know that any given national income will be taxed (there are exceptions!)

$$T = aY - b \tag{5}$$

□ where *T* stands for tax, and *Y* is national income.

☐ This means that the income that households have to spend is no longer *Y* but rather (income less tax), which is called disposable income, Y_d. Hence the consumption function is:

$$C = aY_d + b \tag{6}$$

☐ Previously, the investment, *I* , was taken to be constant.

☐ It is more realistic to assume that planned investment depends on the rate of interest, *r* . As the interest rate rises, so investment falls and we have a relationship:

$$I = cr + d \tag{7}$$

☐ where *c<0* and *d>0*.

Example 2

□ Find the equilibrium level of income and consumption if the consumption function *C=0.4Y+20* is and planned investment *I=10*.

Solution 2

We know that

$$Y = C + I \qquad \text{(from theory)}$$

$$C = 0.4Y + 20 \qquad \text{(given in the problem)}$$

$$I = 10 \qquad \text{(given in the problem)}$$

If the value of I is substituted into the first equation then,

$$Y = C + 10$$

The expression for C can also be substituted to give:

$$Y = 0.4Y + 20 + 10$$

$$Y = 0.4Y + 30$$

$$0.6Y = 30 \qquad \text{(subtract } 0.4Y \text{ from both sides)}$$

$$Y = 50 \quad \text{(divide both sides by 0.6)}$$

The corresponding value of C can be deduced by putting this level of income into the consumption to get:

$$C = 0.4x50 + 20$$

$$= 40$$

Practice 3 (for students)

☐ Find the equilibrium level of income and consumption if the consumption function *C=0.2Y+15* is and planned investment *I=9*.

Exercises

1. If the consumption function is given by

   $$C = 0.7Y + 40$$

   state the values of

   (a) autonomous consumption
   (b) marginal propensity to consume.

   Transpose this formula to express Y in terms of C and hence find the value of Y when $C = 110$.

2. Write down expressions for the savings function given that the consumption function is

   (a) $C = 0.9Y + 72$ (b) $C = 0.8Y + 100$

3. For a closed economy with no government intervention the consumption function is

$$C = 0.6Y + 30$$

and planned investment is

$$I = 100$$

Calculate the equilibrium level of

(a) national income

(b) consumption

4. A consumption function is given by $C = aY + b$.

It is known that when $Y = 10$, the value of C is 28, and that when $Y = 30$, the value of C is 44.

By solving a pair of simultaneous equations, find the values of a and b

Determine the equilibrium level of income when planned investment $I = 13$.

5. Given that

a) $G = 50$

   $I = 40$

   $C = 0.75Y_d + 45$

   $T = 0.2Y + 80$

calculate the equilibrium level of national income.

b) $G=20$
   $I=35$
   $C=0.9Y_d+70$
   $T=0.2Y+25$

6. Consider the national income model

$$Y = C + I$$

$$C = aY_d + 50$$

$$I = 24$$

$$Y_d = Y - T$$

$$T = 20$$

Show that the equilibrium level of national income is given by

$$Y = \frac{74 - 20a}{1 - a}$$

Hence, or otherwise, find the value of a for which $Y = 155$ and find the value of C.

7.

Consider the national income model:

$$Y = C + I$$
$$C = aY_d + 40$$
$$I = 50$$
$$Y_d = Y - T$$
$$T = 30$$

Show that the equilibrium level of national income is given by:

$$Y = \frac{90 - 30a}{1 - a}$$

Also, find the value of *a* for which *Y=180*, and find the value of *C*.

Thank you for attention!