Mathematics for Business: Lecture Notes - 9

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# Compound Interest (Cont.)

The type of compounding which the interest is added on with increasing frequency is called **continuous compounding**. The future value, , of a principal, , compounded continuously for  years at an annual rate of % is calculated by the following formula:



Where is the number 2.718281828459045......

**Exercise -1**: A principal of £1000 is invested 12% interest compounded continuously for 2 years. Calculate the future value?

**Solution-1**: 

**Practice -1**: A principal of £25000 is invested 15% interest compounded continuously for 5 years. Calculate the future value?

**Practice -2**: A principal of £100000 is invested 17% interest compounded continuously for 3 years. Calculate the future value?

**Practice-3**: A principal, £100 is invested at 7% interest for 2 years. Determine the future value if the interest is compounded

a) annually b)semi-annually c) quarterly d)monthly e) weekly f) daily

Also determine the future value of £100 invested at 7% interest compounded continuously for 2 years. Comment about your findings.

Given that there are so many ways of calculating compound interest; people often find it difficult to appraise different investment opportunities. What is needed is a standard ‘benchmark’ that enables an individual to compare different forms of savings or credit schemes on an equal basis. The one that is commonly used is annual compounding. All firms offering investment or loan facilities are required to provide the effective annual rate. This is often referred to as the **annual percentage rate**, which is abbreviated to **APR**. The APR is the rate of interest which, when compounded annually, produces the same yields as the nominal (that is, the stated) rate of interest.

**Exercise -2**: Determine the annual percentage rate of interest of a deposit account that has a nominal rate of 5% compounded monthly?

**Solution-2:** The APR is the overall rate of interest, which can be calculated using scale factors. If the account offers a return of 5% compounded monthly then each month the interest is:



So, about 42% of the amount invested at the beginning of that month. The monthly scale factor is



So in a whole year the principal gets multiplied by



Which can be written as

so the APR is 5.16%

**Practice-4** : Determine the APR if the nominal rate is 10% compounded quarterly.

**Exercise -3**: A firm decides to increase output at a constant rate from its current level of 100000 to 140000 during next 3 years. Calculate the annual rate of increase required to achieve this growth.

**Solution-3**: If the rate of increase is  then scale factor is so, after 3 years, output will be: . To achieve a final output of 140000, the value of is chosen to satisfy the equation



 (divide both side by 100000)

(1+ (take the third root of both sides)

 so, r=12%

**Practice 5**. Exercise 3.2 all questions.

# Investment Appraisal

We have seen the formulas and  for the calculation of compound interest problems. The first formula can be applied to any type of compounding in which the interest is added on to the investment at the end of discrete time intervals. The second formula is used when the interest is added on continuously. As you can clearly see both formulas involve the variables as principal, future value, interest rate and time respectively.

In the case of discrete compounding, the letterrepresent the number of time periods. For continuous compounding,  is measured in years. Given any three of these variables it is possible to work out the value of the remaining variable.

Of particular interest is the case where and  are given, and is the unknown to be determined. In this situation we know the future value, and we want to work backwards to calculate the original principal. This process is called **discounting** and the principal, , is called the **present value**. The rate of interest is sometimes referred to as the **discount rate**.

The two equations above can easily be arranged to produce explicit formulas for the present value under discrete and continuous compounding:

 and similarly,

 (Reciprocals are denoted by negative powers)

**Exercise- 3** Find the present value of £500 in 5 years if the discount rate is 6% compounded

1. Semi-annually
2. Continuously

**Solution-3** :

1. The discount formula for discrete compounding is



If compounding occurs semi-annually then since the interest rate per 6-months is , and since there are ten 6-months period in 5 years. We are given that the future value is £500, so

=373.13

1. The discount formula for continuous compounding is,

, in this formula, is the annual discount rate, which is 6, and  is measured in years so is 5. Hence the present value is:

=370.37

Present values are a useful way of appraising investment projects. Suppose that you are invited to invest £300 today in a business venture that is certain to produce a return of £500 in 5 years’ time. If the discount rate is 6% compounded semi-annually then part (a) of the exercise-3 shows that the present value of this return is 373.13. This exceeds the initial outlay of £300, so the venture is regarded as profitable. We quantify this profit by calculating the difference between the present value of the revenue and the present value of the costs, which is known as the **net present value (NPV).** In this example, the net present value is:

373.13-300=£73.13

Quite generally, a project is considered worthwhile when the NPV is positive. Moreover, if a decision is to be made between two different projects then the one with the higher NPV is the preferred choice.

# Practice -6. Find the present value of £10000 in 15 years’ time if the discount rate is 10% compounded

1. annually
2. semi-annually
3. monthly
4. continuously

**Practice – 7** A projects requiring an initial outlay of £ 20000 is guaranteed to produce a return of £25000 in 2 years’ time. Use the NPV method to decide whether this investment is worthwhile if the prevailing market rate is 6% compounded annually. Would your decision be affected if the interest rate were 15%?

An alternative way of assessing individual projects is based on the **internal rate of return (IRR).** This is the annual rate which, when applied to the initial outlay, yields the same return as the project after the same number of years. The investment is considered worthwhile provided the IRR exceeds the market rate. Obviously, in practice, other factors such as risk need to be considered before a decision is made.

Let us use **Practice-7** to illustrate IRR method.

To calculate the IRR we must use the formula,



We are given =25000, =20000, =2, so we need to solve:



1.25=

1.12=

=12 and so the IRR is 12%. The project therefore to be recommended because this value exceeds the market rate of 6%.

**Practice-8**: An investment project requires an initial outlay of £16000 and will produce a return of £22000 at the end of 4 years. Use the

1. net present value
2. internal rate of return

methods to decide whether this is worthwhile if the capital could be invested elsewhere at 14% compounded annually?

**Practice – 9**: You are given the opportunity of investing in one of three projects. Projects X, Y and Z. which require initial outlays of £10000, £50000, and £75000 for 4 years, respectively. Which of these three projects would you prefer to invest in if the market rate is 10% compounded annually?

# Geometric Series

All of the problems we considered so far involved a single lump-sum payment into an investment account. The task was simply to determine its future value after a period of time when it is subject to a certain type of compounding. Here, we are going to extend this to include multiple payments. This situation occurs whenever individuals save regularly or when businesses take out a loan that is paid back using fixed monthly or annual instalments. To tackle these problems we need to be able to sum consecutive terms of a geometric progression. Such an expression is called a **geometric series**. A geometric series where  is the first term and is the geometric ratio is calculated as follow:

