Mathematics for Business: Lecture Notes -1

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# 1. Introduction to Algebra

In Mathematics numbers are classified into one of three types: positive, negative or zero. When using the four operations in math (addition, subtraction, multiplication and division) rule varies for negative numbers. As such, negative number is interpreted based on the case. For instance, -4 on the temperature on a thermometer scale measured in degrees centigrade is interpreted a degree of 4 below freezing (=zero). On the other hand, -£100,000 signifies a loss of/ or a debt of a £100.000 of firms or personal finance, and so on. (Discuss similar cases). Perhaps, the easiest way to illustrate the three types of integer is a number line (see. Figure 1.1).

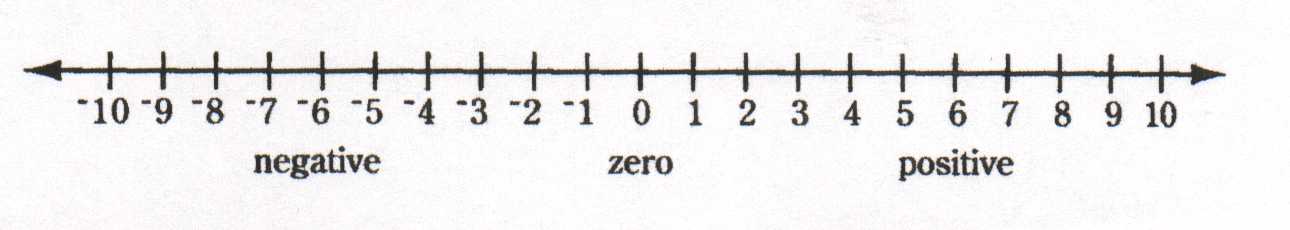


Figure 1.1 Number Line

The line continues left and right forever.

The rules for the multiplication of negative numbers are:

NEGATIVE x NEGATIVE =POSITIVE

NEGATIVE x POSITIVE = NEGATIVE

It does not matter in which order two numbers are multiplied, so

POSITIVE x NEGATIVE = NEGATIVE

The rules produce:

(-7) x (-5) = 35

(-3) x (4) = -12

(6) x (-3) =-18

respectively. The same rules apply for division, since division is the same sort of operation as multiplication. Thus, exactly the same rules operate when one number is divided by another. For instance,

(-30)  (-10)= 3

(-45)  (9)= -5

(52)  (-13) =-4

To avoid complexity when multiplying or dividing multiple numbers, it is perhaps simplest to ignore the signs to begin with and just to work the answers out. The final result is negative if the total number of minus signs is odd and positive if the total number is even.

**Example (1)** : Evaluate

1. (-2)x(-3)x (4)x (-1)= b) =

**Solution (1)** :

1. Ignoring the signs gives

2x3x4x1=24

There are an odd number of minus signs (in fact, three) so the answer is -24

1. Ignoring the signs gives

==10

There are an even number of minus signs (in fact, four) so the answer is 10

**Practice Problem (1)**: Evaluate

a)5x(-6)= b) (-1)x(-2)= c)(-50) 10=

d)(-5) (-1)= e)2x(-1)x(-3)x6 = f) =

To add or subtract negative numbers it helps to think in terms of “money/debt”:

Assume you have £100 in your pocket, but £200 debt, than you have £ 100 debt left. The money you have has a positive sign, whereas the debt you have has a negative sign. So if you have more money than your debt, you will have some of your money left; but if your debt is more than the money you have you will left off with some of your debt. For example:

100 – 200 = -100 (your debt is greater than your money)

-50 + 150 = 100 (your money is greater than your debt)

-130 – 30 = -160 (you have debt and then more debt, your debt gets even more)

**Example (2)** : Evaluate

1. -3+4= b) -2-8= c)-1-4-5= d)-4-5+6+1= e)-1+7-3+4=

**Solution (2)**

1. 1 b)-10 c)-10 d)-2 e)7

On the other hand, assume *a* and *b* are positive numbers, *a*-(-*b*) is taken to be *a*+*b*. This follows from the rule for multiplying two negative numbers, since -(-*b*)= (-1)x(-*b*) = *b*

**Example (3)**: Evaluate

1. 3-(-4)= b)-2-(-6)= c) –(-5)+3= d)-(-2)-6= e)-(-1)-(-7)=

**Solution (3)**:

1. 3+4=7 b)-2+6=4 c)5+3=8 d)2-6=-4 e)1+7=8

**Practice Problem (2)**: Evaluate

1. 40-(-32)+5= b)-(-4)+(-7)-(-3)= c)-12+4-(-5)= d)5-15+3-(-4)=

## Expressions

In algebra, letters are used to represent numbers. In pure mathematics, the most common letters used are *x* and *y*. However, in applications it is helpful to choose letters that are more meaningful, so we might use *Q* for quantity and *I* for investment. An algebraic expression is then simply a combination of these letters, brackets and other mathematical symbols such as + or -. For example, the expression



Can be used to work out how money in a saving account grows over a period of time. The letters *P*, *r* and *n* represents the original sum invested (called the principal- hence the use of the letter *P*) , the rate of interest and the number of years respectively. To work it all out, you not only need to replace these letters by actual numbers, but also need to understand the various conventions that go with algebraic expressions such as this.

In algebra when we multiply two numbers represented by letters we usually supress the multiplication sign between them. The product of *a* and *b* would simply be written as *ab* without bothering to put the multiplication sign between the symbols. Likewise when a number represented by *Y* is doubled we write *2Y*. Here are some examples:

*P*x*Q* is written as *PQ*

*d*x*8* is written as *8d*

*n*x*6*x*t* is written as *6nt*

*z* x*z* is written *z2*

*1* x *t* is written as *t*

In order to evaluate these expressions it is necessary to be given the numerical value of each letter. Once this has been done you can work out the final value by performing the operations in the following order:

Brackets first **(B)**

Indices second **(I)**

Division and Multiplication **(DM)**

Addition and Subtraction **(AS)**

This is sometimes remembered using the acronym **BIDMAS** and it is essential that this ordering is used for working out all mathematical calculations. For example, suppose you wish to evaluate each of the following expressions when x= 2:

3x2 and (3x)2

Substituting x=2 into the first expression gives

3x2=3x22 (the multiplication sign is revealed when we switch from algebra to numbers)

=3x4

=12

whereas in the second expression we get

(3x)2=(3x2)2(again the multiplication sign is revealed)

=(6)2

=36

The two answers are not the same so the order indicated by **BIDMAS** does really matters.

**Example 4**: Evaluate

1. Find the value of 5x-2y when x=5 and y=1
2. Find the value of 3x2-5x+4 when x=4
3. Find the value of -3(P)2+3P+3 when P=3
4. Find the value of (4-A)-(-4A+7) when A=6

**Solution 4**:

1. 5x-2y=5x5-2x1 (substituting numbers)

=25-2 (multiplication has priority over subtraction)

=23

1. 3x2-5x+4 =3x42-5x4+4 (substituting numbers)

=3x16-20+4 (indices have priority over multiplication and addition/subtraction)

=48-20+4 (multiplication has priority over addition/subtraction)

=32

1. -3(P)2+3P+3 =-3(3)2+3x3+3 (substituting numbers)

=-3x9+ 3x3 +3 (brackets has priority over multiplication and addition)

=-27+9+3 ((multiplication has priority over addition/subtraction)

=-15

1. (4-A)-(-4A+7) =(4-6)-(-4x6+7) (substituting numbers)

=(-2)- (-24+7) (brackets first)

=-2-(-17) (multiplication rules)

=-2+17 =15

**Practice Problem 3**: Evaluate each of the following by replacing the letters by the given numbers:

1. 3Q+4 when Q=4
2. 2A2B+B when A=2 and B=3
3. x2y+2xy2 when x=-1 and y=3

**Like terms** are multiples of the same letters (or letters). For example, 3*a*, -4*a*, 6*a*, -*a* are all multiples of *a* and so are like terms. If an algebraic expression contains like terms which are added or subtracted together then it can be simplified to produce an equivalent shorter expression. For example:

1. a+4a-2a=3a
2. x2+3x2+7x2=11x2
3. -3Q+4Q+Q-2Q=0

## Brackets

It is useful to be able to take an expression containing brackets and rewrite it as an equivalent expression without brackets and vice versa. The process of removing brackets is called expanding brackets or multiplying out brackets. This is based on the distributive law, which states that for any three numbers x, y and z,

x(y+z)= xy+xz

For example:

a)5(2+4)=5x6=30 also, 5(2+4)=5x2+5x4=10+20=30

b)x(y+z+t)= xy+xz+xt

**Example 5**: Multiply out the brackets, simplifying your answers as far as possible.

a)3(x-4)=

b)(4-2a)a=

c)x+2y+3-(2x-2y+3)=

d)2x(3-x)+3x2=

**Solution 5:**

a) 3(x-4)=3x-12

b) (4-2a)a=4a-2a2

c) x+2y+3-(x-2y+3)= x+2y+3-2x+2y-3

= -x+4y (collecting alike terms)

d)2x(3-x)+3x2=6x-2x2+3x2

=6x+x2 (collecting alike terms)