Mathematics for Business: Lecture Notes - 4

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# 4. Algebraic solution of simultaneous linear equations

In this lecture, we are going to cover **elimination method** while searching for solution to simultaneous linear equations. In this method, in order to find the unknowns we will eliminate one (or more) unknowns during the process. Let us illustrate this method by one simple example:

**Example 1**: Solve the system of the equations:

2*x*+3*y*=11

2*x*+*y*=5

**Solution 1**: Let us eliminate *x*’s first.

We can do this by subtracting second equation from the first equation.

2*x*+3*y*= 11

-2*x*-*y* = -5 (*x*’s cancel)  2*y*=6  *y*=3

Substitute the value of y=3 in any original equation to deduce x:

2*x*+3(3)=11  2*x*+9=11  2*x*=2  *x*=1

The original system of equation **has a unique solution**, which is (1,3).

**Example 2**: Solve the system of the equations:

*x*-2y=1

2*x*-4*y*=-1

**Solution 2**: The variable *x* can be eliminated by doubling the first equation and subtracting the second:

2*x*-4*y*=2

- 2*x*-4*y*=-1

0=5 (both the *x*’s and *y*’s cancel)

Clearly the statement “0=5” makes no sense. In this case it can be said that the original system of equations **does not have a solution.** (these lines are parallel)

**Example 3**: Solve the system of the equations:

2*x*-4*y*=1

5*x*-10*y*=5/2

**Solution 3**: The variable x can be eliminated by multiplying the first equation by 5, multiplying the second equation by 2 and subtracting,

10*x*-20*y*=5

- 10*x*-20*y*=5

0=0 (everything cancels including the right-hand side)

This particular system of equations has **infinitely many solutions**.( since these equations represent the same line). In other words, any point lying on either of the line will be the solution.

**Practice 1:** Solve the system of the equations:

a) 3*x*-6*y*=-2 b) -5*x*+*y*= 4 c)2*x*+6*y*=8 d)4*x*-2*y*=10

-4*x*+8*y*=-1 10*x*-2*y*=-8 3*x*-6*y*=4 *x*+2*y*=1

e)-2*x*+*y*=2 f) 3*x*+4*y*=12 g)2*x*+*y*=4 h) *x*+*y*=1

2*x*+*y*=-6 *x*+4*y*=8 4*x*-3*y*=3 6*x*+5*y*=15

**Practice 2**: Exercises 1.4, questions 1, 3, 4. Exercises 1.4\* questions 1, 2.

# 5. Supply and Demand Analysis

Microeconomics is concerned with the analysis of the economic theory and policy of individual firms and markets. In this lecture, we will be focusing on one particular aspect known as **market equilibrium**, in which the supply and demand balance. We will be using some algebra to be able to calculate the equilibrium price (P) and quantity (Q). However, before we do this it is useful to explain the concept of a function. This idea is central to nearly all applications of mathematics in economics.

A **function**, ,is a rule which assigns to each incoming number, , a uniquely defined outgoing number, . You can think of a function as a factory/ or a machine that performs a dedicated arithmetic calculation. As an example, consider the rule ‘triple and subtract 1’. Let us see the effect of this rule on few specific numbers in Figure 1:

2 3x2-1=5

Triple and subtract

*INCOMING* *OUTGOING*

Triple and subtract

-4 3x(-4)-1=-13

Triple and subtract

5 3x5-1=14

Figure 1

There are, two alternative ways of expressing this rule which are more concise. We can write either

 or 

The first expression of these is familiar to you from our previous work; corresponding to any incoming number, , the right-hand side tells you what to do with to generate the outgoing number, .

The second notation is also useful. It has the advantage that it involves the label , which is used to name the rule. This notation also enables the information conveyed in Figure 1 to be written :

, (it is read as ‘ of 2 equals 5’)

 (it is read as ‘of -4 equals -13’)

and finally  (it is read as ‘ of 5 equals 14’)

**Example 4**: If , find the value of

a) and b)

**Solution 4**:

a)Substituting  into  gives;

= 8+2-2=8

b) Substituting  into  gives;

=-27-3-2=-32

**Practice 3**: Evaluate

a) b) c) d) e) f)

for the two functions

and 

Do you notice any connection between and ?

The incoming and outgoing variables are referred to as the **independent** and **dependent** variables, respectively.

The value of clearly ‘depends’ on the actual value of  that is fed into the function.

For example, in microeconomics the quantity demanded, , of a good depends on the market price, . We might express this as:



Such a function is called a **demand** function.

The two functions, and , are said to be **inverse function**: that is if  is the inverse of and, equivalently, is the inverse of .

When finding out an inverse function of a given function, we should undo each step in reverse order. Remember **BIDMAS** while doing so.

**Example 5**: Find the inverse of given functions below:

a) b) c)

**Solution 5**:

a) (multiply by 3, subtract 1)

 (divide by 3, add 1)

b)  (divide by 2, multiply by 3, subtract 4)

 (multiply by 2, divide by 3, add 4)

c)  (multiply by -4, add 5)

 (divide by -4, subtract 5)

**Practice 4**: Find the inverse of given functions below:

a)

b)

c)

## Building a Model

Any linear function is given in the form of:

.

where and are constant **parameters**.

is said to be a **decreasing function** if  (is less than zero),

is said to be an **increasing function** if  ( is greater than zero)

To be able to build a model, we need at least two points which passes through the line defined by -function.

**Example 6**: If a linear function  gets a value of 5 when , and value of 13 when , find the value of ?

**Solution 6**: First, we should evaluate .

To do that, let us substitute all given values as follows,

 (I)

 (II)

we now have two sets of equations, I and II. First eliminate  , by subtracting second equation from the first one:





(’ cancelled)  
by substituting into any given equation we find 



So, the function is . Now we can evaluate the value of :



**Practice 5**: If a linear function  gets a value of 1 when and get the value of 1 when , find the value of .