Mathematics for Business: Lecture Notes - 7

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# Mathematics of Finance

## Percentages

In order to be able to handle financial calculations, it is necessary to use percentages proficiently. The word ‘percentage’ literally mean ‘per cent’, i.e. per hundredth, so that whenever we speak of r% of something, we simply mean the fraction (r/100) ths of it.

For example;

25% is the same as 

30% is the same as 

50% is the same as 

**Practice 1**. Calculate

1. 10% of 130
2. 56% of 14
3. 95% of 50
4. 15% of £150.00
5. 20% of $1,000.00
6. 35% of £1.50

Whenever any numerical quantity increases or decreases, it is customary to refer this change in percentage terms.

**Exercise 1**.

1. An investment rises from £2500 to £3375. Express the increase as apercentage of the original.
2. At the beginning of a year,the population of a small village is 8400. If the annual rise in population is 12%, find the population at the end of the year.
3. In a sale, all prices are reduced by 20%. Find the sale of a goodoriginally costing £580.

**Solution 1**.

1. The rise in the value of investment is 3375-2500=875

As a fraction of theoriginal this is

 so the percentage rise is 35%.

1. As a fraction

12% is the same as 

so the rise in population is 0.12x8400=1008

Hence the final population is 8400+1008=9408

1. As a fraction

20% is the same as 

So the fall in price is

0.2x580=116

Hence the final price is 580-116= £464

**Practice 2**.

1. A firm’s annual sale rise from 50 000 to 55 000 from one year to the next. Express the rise as a percentage of the original.
2. The goverment imposes a 15% tax on the price of a good. How much does the consumer payfor a good priced by a firm at £1360.
3. Investments fall during the course of a year by 7%. Find the value of an investment at the end of the year if it was worth £9500 at the beginning of the year.

## Scale Factor

In the previous examples, the calculations were performed in two seperate stages. The actual rise or fall was firts worked out, and these changes were then applied to the original value to obtain the final answer. It is possible to abtain this answer in a single calculation, and we now describe how this can be done.

To be spesific, let us suppose that the price of good is set to rise by 9%, and its current price is £78. The new price consists of the original ( which can be thought of as 100% of the £78) plus the increase (which is 9% of £78).

The final price is therefore

100%+9%=109%(of the £78)

which is the same as 

In other words, in order to calculate the final price all wehave to doisto multiply by the **scale factor**,1.09. Hence the new price,

1.09x78=£85.02

One advantage of this approach is that it is then just as easy go backwards and work out the original price from the new price. To go backwards in time we simply divide by the scale factor. For example, if the final price of a good is £1068.20 then before 9% increase the price would have been

10681.09=980

In general, if the percentage rise is r% then the final value consists of the original (100%) together with the increase (r%), giving a total of



To go forwards in time we multiply by this scale factor, whereas to go backwards we divide.

**Exercise 2** –

1. If the annual rate of inflation is 4%, find the price of a good at the and of a year if its price at the beginning of the year is £25.
2. The cost of a good is £750 including a sales tax of %20. What is the cost excluding the sales tax?
3. Express the rise from 950 to 1007 as a percentage.

**Solution 2**-

1. The scale factor is , so price after the increase 25x1.04= £26
2. The scale factor is , price before the increase 750=£25
3. The scale factor is

 which can be thought of as so the rise is 6%.

It is possible to use scale factors to solve problems involving percentage decreases. To be spesific, suppose that an investment of £76 falls by %20. The new value is the original (100%) less the decrease (20%), so is 80% of the original. The scale factore therefore0.8, giving a new value of 0.8 x 76 =£60.80.

In general, the scale factor for an r% decrease is,



Once again, you multiply by this scale factor when going forwards in time and divide when going backwards.

**Exercise 3**-

1. The value of a car depriciates by 25% in a year. What will a car, currently priced at £43 000, be worth in a year’s time?
2. After a 15% reduction in a sale, the price of a good is £39.95. What was the price before the sale began?
3. The number of passengers using a rail link fell from 190 205 to 174 989. Find the percentage decrease.

**Solution 3**-

1. The scale factor is, , so the new price is 43 000x0.75= £32 250 (forwards in time)
2. The scale factor is, , so the original price was 39.950.85=£47 (backwards in time)
3. The scale factor is,  which can be thought of as , so the fall is 8%.

The final application of scale factors that we consider is to the calculation of overall percentage changes. This can be done by simply multiplying together successive scale factors.

**Exercise 4** –

1. Share prices rise by 32% during the first halfof the year and rise by a further 10% during the second half. What is the overall percentage change?
2. Find the overall percentage change in the price of a good if it rises by 5% in a year but is then reduced by 30% in a sale.

**Solution 4**-

1. , and 

The net effect is to multiply by their product

1.32 x1.1 =1.452

, so the overall change is 45.2%

1. The individual scale factors are 1.05 and 0.7, so the overall scale factor is

1.05 x 0.7 = 0.735 (<1 so it is a decrease)



we see that this scale factor represents a 26% decrease.

# Index Numbers

Economic data often take the form of a time series; values of economic indicators are available on an annual, quarterly or monthly basis, and we are interested in analysing the rise and fall of these numbers over time. Index numbers enable us to identfy trends and relationshis in the data. The following example shows you how you calculate index numbersand how to interpret them.

**Example 5**- Table below shows the values of household spending ( in billions of pounds) during a 5-year period. Calculate the index numbers when 2000 is taken as the base year and give a brief interpretation.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Year** | | | | |
| 1999 | 2000 | 2001 | 2002 | 2003 |
| **Household spending** | 689.9 | 697.2 | 723.7 | 716.6 | 734.5 |

**Solution 5**- When finding index numbers, a base year is chosen and the value of 100 is allocated to that year. So,in this example, the index number of 2000 is 100.

To find the index number of the year 2001 we work out the scale factor associated with the change in household spending from the base year, 2000 to 2001, and then multiply the answer by 100.

for the year 1999 :

(value of household spending in 1999 was 98.5% of its value in 2000)

for the year 2001:

(value of household spending in 2001 was 103.8% of its value in 2000)

for the year 2002:

(value of household spending in 2002 was 102.8% of its value in 2000)

for the year 2003:

(value of household spending in 2003 was 105.3% of its value in 2000)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Year** | | | | |
| 1999 | 2000 | 2001 | 2002 | 2003 |
| **Household spending** | 689.9 | 697.2 | 723.7 | 716.6 | 734.5 |
| **Index number (2000)** | 98.5 | 100 | 103.8 | 102.8 | 105.3 |

**Practice 3** . Find the index numbers of each share price shown in Table below, taking April as the base month. Compare the performances of these two shares during this period. (Assume investing £1000 for share A and B seperately in January)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Month** | **Jan** | **Feb** | **Mar** | **Apr** | **May** | **Jun** | **Jul** | **Aug** |
| **Share A** | 0.31 | 0.28 | 0.31 | 0.34 | 0.40 | 0.39 | 0.45 | 0.52 |
| **Share B** | 6.34 | 6.40 | 6.45 | 6.52 | 6.57 | 6.43 | 6.65 | 7.00 |

# Inflation

Over a period of time, the prices of many goods and services usually increase. The annual rate of inflation is the average percentage change in a given selection of these goods and services, over the previous year. Seasonal variations are taken into account, and the particular basket of goods and services is changed periodically to reflect changing patterns of household expenditure. The presence of inflation is particularly irritating when trying to interpret a time series that involves a monetary value. It isinevitable that this will be influenced by inflation during any year, what is of interest is the fluctuation of a time series ‘over and above’ inflation. Economists deal with this by distinguishing between nominal and real data.

**Nominal data** are the original, raw data. These are based on the prices that prevailed at the time.

**Real data**, are the values that have been adjusted to take inflation into account. The standard way of doing this is to picka year and then convert the values for all other years to the level that they would have had in this base year.

**Example -6** Table below shows the price( in thousands of pounds) of an average house in a certain town during a 5-year period. The price quoted is the value of the house at the end of each year. Use the annual rates of inflation given in table below to adjust the prices to those prevailing at the end of 1991. Compare the rise in both the nominal and real values of house prices during this period.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Year** | | | | |
| **1990** | **1991** | **1992** | **1993** | **1994** |
| **Average house price** | 72 | 89 | 93 | 100 | 106 |
| **Annual rate of inflation** | - | 10.7% | 7.1% | 3.5% | 2.3% |

**Solution -6** :

The base year is 1991.

The value of the house at the end of 1991 is obviously £89 000, since no adjustment need to be made.

At the end of 1992, it is worth £93 000. However, during that year inflation year was 7.1%. To adjust this price to ‘1991 prices’ we simply divide by the scale factor.1.071 (=)

(we divide it, since we are going backwards in time)

To adjust the price of the house in 1993 we first need to divide by 1.035(= ) to backtrack totheyear 1992,and then divide again by 1.071 to reach 1991.



Similarly, for the 1994 year, the adjusted value is:



And for 1990, the adjusted value is,

72000x1.107=79704 (going forward in time so multiply)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Year** | | | | |
| **1990** | **1991** | **1992** | **1993** | **1994** |
| **Nominal house price** | 72 | 89 | 93 | 100 | 106 |
| **1991 house price** | 80 | 89 | 87 | 90 | 93 |

Table above presents the nominal and the ‘constant 1991’ values of the house (rounded to the nearest thousands) for comparison. As can be seen from the table, apart from the gain during 1991, the increase in value has, in fact, been quite modest.

**Practice 4**: Questions from 1 to 13 from Exercise 3.1