Mathematics for Business: Lecture Notes - 8

Dr. Cansu Unver Erbas|cnsunvr@gmail.com

# Simple Interest

Simple interest is only used on short-term notes- often of duration less than 1 year. The concept of simple interest, however, forms the basis of much of the rest of the material developed in this lecture, for which time periods may be much longer than a year.

If you deposit a sum of money in a savings account or if you borrow a sum of money  from a lending agent, then  is referred to as the principal. When money is borrowed- whether it is a savings institution borrowing from you when you deposit money in your account or you borrowing from a lending agent- a fee is charged for the money borrowed. This fee is rent paid for the use of another’s money, just as rent is paid for the use of another’s house. The fee is called interest. It is usually computed as a percentage ( called the interest rate) of the principal over a given period of time. The interest rate, unless otherwise stated, is an annual rate. Simple interest is given by the following formula:

Simple interest ; 

where

: principal

: annual sample interest rate (written as a decimal)

:time in years

**Example 1**. Calculate an interest on a loan of $100 at 12% for 9 months.

**Solution 1**:



=9

At the end of 9 months, the borrower would repay the principal ($100) plus the interest ($9), or a total of $109.

# Amount : Simple Interest



where

: principal

: annual sample interest rate (written as a decimal)

:time in years

: amount, or future value

**Example 2**: Find the total amount due on a loan of $800 at 9% simple interest at the end of 4 months.

**Solution 2**:

=800, =0.09, year, thus

=824

**Practice 1**: If you want to earn an annual rate of 10% on your investments, how much (to the nearest cent) should you pay for a note that will be worth 5000 in 9 months?

**Practice 2**: T-bills (treasury bills) are one of the instruments the U.S. Treasury Department uses to finance the public debt. If you buy a 180-day T-bill with a maturity value of $10,000 for $9,693.78, what annual simple interest rate will you earn? (express the answer as a percentage, up to 2 decimal places)

**Practice 3**: A loan of $7250 was repaid at the end of 8 months. What size repayment checks (principal and interest) was written, if a 9% annual rate of interest was charged?

**Practice 4**: If you paid $120 to a loan company for the use of $2000 for 90 days, what annual rate of interest did they charge?

# Compound Interest

If at the end of a payment period the interest due is reinvested at the same rate, then the interest as well as the original principal will earn interest during the next payment period. Interest paid on interest reinvested is called compound interest.

**Example 3:** Suppose you deposit $1000 in a bank that pays 8% compounded quarterly. How much will the bank owe you at the end of a year?

**Solution 3**: Compounded quarterly means that earned interest is paid to your account at the end of each 3-month period and that interest as well as the principal earns interest for the next quarter.

The amount in the account at the end of the first quarter after interest has been paid:



Now, $1020 is your new principal for the second quarter. At the end of the second quarter, after interest is paid, the account will have:



Similarly, at the end of the third quarter, you will have :



Finally at the end of the fourth quarter, the account will have:



Let us look over the calculations:

 end of first quarter

 end of second quarter

 end of third quarter

 end of fourth quarter

It appears that at the end of quarters, we would have

 end of th quarter

In summarise, compounded interest is calculated based on the general formula below:



Where  and

 :annual interest rate

 :number of compounding periods per year.

 :rate per compounding period

 :total number of compounding periods.

 :principal( or present value)

 : amount (future value) at the end of periods.

**Example 4**: If $1000 is invested at 8% compounded

a)annually b)semi-annually c)quarterly d)monthly

what is the amount after 5 years?

**Solutions 4**:

a)Compounding annually means there is one interest payment period per year. Thus, =5, 



b) Compounding semi-annually means there are two interest payment periods per year. Thus, the number of payment periods in 5 years is =2x5=10, and the interest rate per period is



So, 

c) Compounding quarterly means there are four interest payment periods per year. Thus, the number of payment periods in 5 years is =4x5=20, and the interest rate per period is

 so,



d) Compounding monthly means there are twelve interest payment periods per year. Thus, the number of payment periods in 5 years is =12x5=60, and the interest rate per period is

 so,



**Practice 5**: How much should you invest now at 10% compounded quarterly to have 8000 toward the purchase of a car in 5 years?

**Practice 6**: A new born child receives a $5000 gift toward a college education from her grandparents. How much will the $5000 be worth in 17 years if it is invested at 7% compounded quarterly?

**Practice 7**: Which is the better investment and why: 9% compounded monthly or 9.3 % compounded annually?

**Practice 8** : If an investment company pays 8% compounded semi-annually, how much should you deposit now to have $6000

1. 3 years from now? B) 6 years from now?