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FACULTY OF ECONOMICS & ADMINISTRATIVE SCIENCES

MATHEMATICS FOR BUSINESS LECTURE NOTES-6

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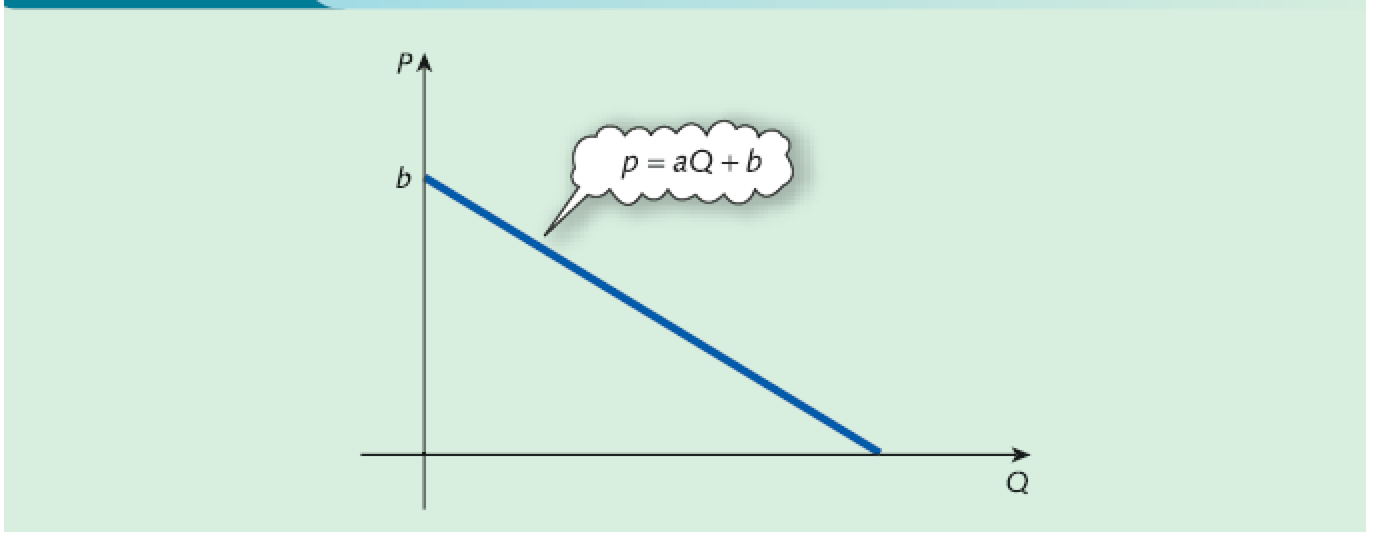
Supply and demand analysis

- Microeconomics is concerned with the analysis of the economic theory and policy of individual firms and markets.
- In this section we focus on one particular aspect known as market equilibrium, in which the supply and demand balance.
- In microeconomics the quantity demanded, *Q,* of a good depends on the market price, *P.* We might express this as $Q = f(P)$.
- Such a function is called a ***demand function.***
- Given any particular formula for *f (P)* it is then a simple matter to produce a picture of the corresponding demand curve on graph paper.

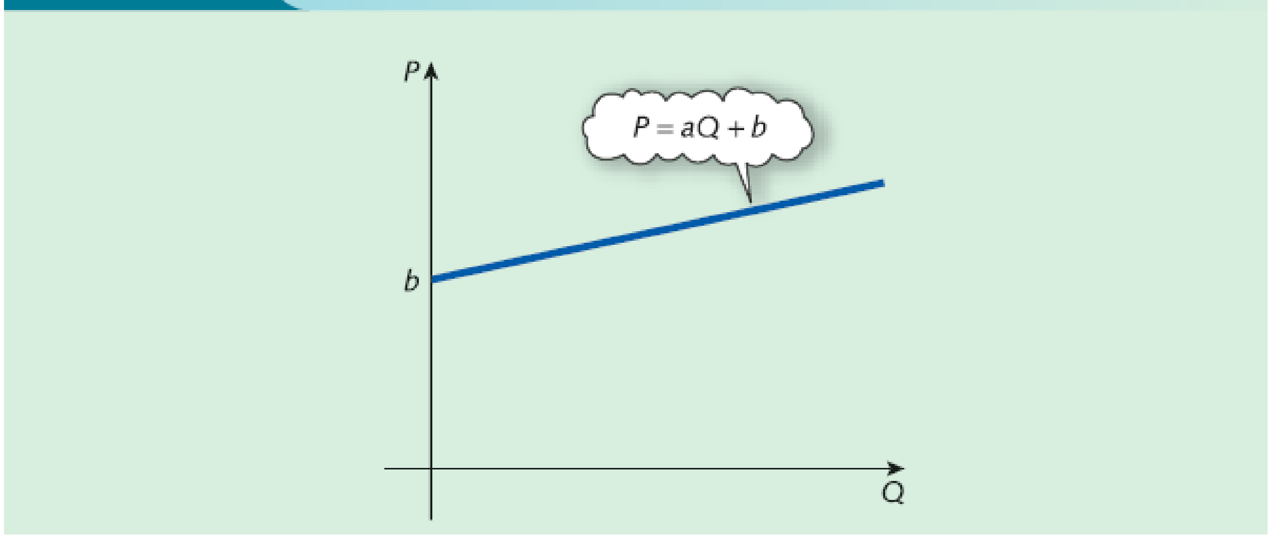
- A graph of a typical linear demand function is shown in Figure 1.14 (overleaf).
- Elementary theory shows that demand usually falls as the price of a good rises and so the slope of the line is negative.
- Mathematically, *P* is then said to be a **decreasing** function of Q.

Figure 1.14



- The ***supply*** function is the relation between the quantity, *Q*, of a good that producers plan to bring to the market and the price, *P*, of the good.
- A typical linear supply curve is indicated in Figure 1.18.
- Economic theory indicates that, as the price rises, so does the supply.
- Mathematically, *P* is then said to be an **increasing** function of *Q*.
- A price increase encourages existing producers to raise output and entices new firms to enter the market.

Figure 1.18



- The line shown in Figure 1.18 has equation

$$P = aQ + b$$

- with slope $a > 0$ and intercept $b > 0$.
- Note that when the market price is equal to b the supply is zero.
- It is only when the price exceeds this threshold level that producers decide that it is worth supplying any good whatsoever.
- It is important to note that this is a simplification of what happens in the real world.
- The supply function does not have to be linear and the quantity supplied, Q , is influenced by things other than price. These exogenous variables include the prices of factors of production (that is land, capital, labour and enterprise), the profits obtainable on alternative goods, and technology.

- Figure 1 shows both supply and demand curves in general:

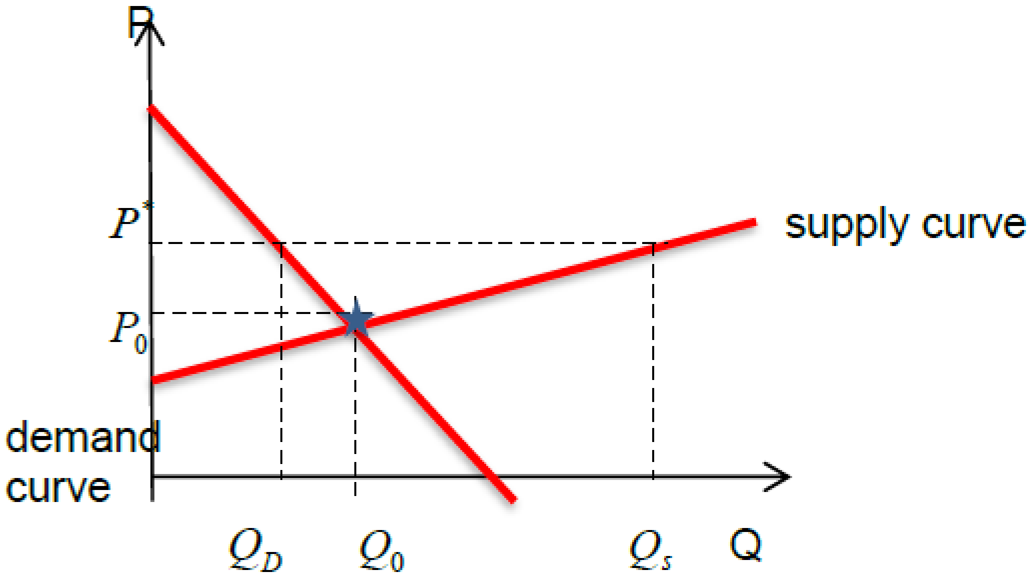


Figure 1

- Of particular significance is the point of intersection. At this point the market is in **equilibrium (★)** because the quantity supplied exactly matches the quantity demanded.
- The corresponding price, P_0 , and the quantity, Q_0 , are called the equilibrium price and quantity.
- In practice, it is often the deviation of the market price away from the equilibrium price that is of most interest.

- Suppose that the market price P^*, exceeds the equilibrium price, P_0 .
- From Figure 1., the quantity supplied, Q_s , is greater than the quantity demanded, Q_D, so there is excess supply.
- There are stocks of unsold goods, which tend to depress prices and cause firms to cut back production. The effect is for ‘market forces’ to shift the market back down toward equilibrium.
- Likewise, if the market price falls below equilibrium price then demand exceeds supply. This shortage pushes prices up and encourages firms to produce more goods, so the market drifts back up towards equilibrium.

Example 1

- The demand and supply function of a good are given by:

$$P = 2Q_D + 60$$

$$P = 3Q_S - 40,$$

- where P, Q_D, Q_S denote the price, quantity demanded and quantity supplied respectively.
  a) Determine the equilibrium price and quantity
  b) Determine the effect on the market equilibrium if the government decides to impose a fixed tax of £1 on each good.

Solution 1

a) In the equilibrium, the quantity demanded will be equal to quantity supplied.

$Q_S = Q_D = Q$ so the set of equation will take the form of:

$2Q + 60 = 3Q - 40$ (solve the simultaneous equations for Q)

Q =100 (equilibrium quantity)

By substituting Q in any given original equation, P is found to be 260. (Equilibrium price)

b) If the government imposes a fixed tax of £1, than the money that the firm actually receives from the sale of each good will be less the tax that is $P-1$.

Replacing P by $P-1$ in supply function, we get the new supply equation:

$P-1 = 3Q_S - 40$, that is

$P = 3Q_S - 39$

In the equilibrium, the quantity demanded will be equal to quantity supplied.

$Q_S = Q_D = Q$ so the new set of equations will take the form of:

$2Q + 60 = 3Q - 39$ (solve the simultaneous equations for Q)

Q =99

By substituting Q in any given original equation, P is found to be 258. (Equilibrium price after a fix tax)

Practice 1 (for students)

- The demand and supply function of a good are given by:

$$P = \frac{Q_D}{2} + 30$$

$$P = Q_S - 10$$

- where P, Q_D, Q_S denote the price, quantity demanded and quantity supplied respectively.
  a) Determine the equilibrium price and quantity
  b) Determine the effect on the market equilibrium if the government decides to impose a fixed tax of £2 on each good.

Practice 2 (for students)

- The demand and supply function of a good are given by:

$$P = Q_D^2 + 2$$

$$P = 3Q_S^2 - 30$$

- where P, Q_D, Q_S denote the price, quantity demanded and quantity supplied respectively.
- Determine the equilibrium price and quantity

- Mathematical modelling involves the use of formulae to represent the relationship between economic variables.
- So far, we have seen how useful supply and demand formulae are. And, we have seen supply-demand formulae as expressed by the relationship between price and quantity (demanded and supplied), and is presented in the form of

$$P = aQ + b \, .$$

- However, if we are given many values of *P*, it is clearly tedious and inefficient for us to solve the equation each time to find *Q*. The preferred approach to transpose the formula for *P*. In other words, we rearrange the formula

P = an expression involving Q

into

Q = an expression involving P

Example 2

- Rearrange the formulae:

a) $Q = -2P + 10$ to express P in terms of Q (make P the subject of given formula)

b) $2Q = \frac{P}{3} - 5$ to express P in terms of Q

c) $Q = 4P^2 - 5$ to express P in terms of Q

d) $Q = \sqrt{\frac{2P+5}{3P-2}}$ to express P in terms of Q

Solution 2

a) $Q = -2P + 10$

$Q - 10 = -2P$ (subtract 10 from both sides)

$\frac{Q-10}{-2} = P$ (divide both side by -2)

$P = -\frac{Q}{2} + 5$ (rearrange)

b) $2Q = \frac{P}{3} - 5$

$2Q + 5 = \frac{P}{3}$ (add 5 to the both side)

$3 \times (2Q + 5) = P$ (multiply both side by 3)

$P = 6Q + 15$ (rearrange)

Solution 2

c) $Q = 4P^2 - 5$

$Q + 5 = 4P^2$ (add 5 to the both side)

$\frac{Q+5}{4} = P^2$ (divide both side by 4)

$\sqrt{\frac{Q+5}{4}} = P$ (take a square root of both sides)

Solution 2

d) $Q=\sqrt{\dfrac{2P+5}{3P-2}}$

$Q^2=\dfrac{2P+5}{3P-2}$ (square both side of the equation)

$Q^2\times(3P-2)=2P+5$ (multiply both sides by $(3P-2)$)

$3PQ^2-2Q^2=2P+5$ (multiply out bracket)

$3PQ^2-2P=2Q^2+5$ (collect P's on one side of the equation)

$P(3Q^2-2)=2Q^2+5$ (take out a common factor of P)

$P=\dfrac{2Q^2+5}{3Q^2-2}$ (divide both side by $3Q^2-2$ to make P the subject of given formula)

Example 3

The demand and supply functions for two interdependent commodities are given by

$$Q_{D_1} = 10 - 2P_1 + P_2$$

$$Q_{D_2} = 5 + 2P_1 - 2P_2$$

$$Q_{S_1} = -3 + 2P_1$$

$$Q_{S_2} = -2 + 3P_2$$

where Q_{D_i}, Q_{S_i} and P_i denote the quantity demanded, quantity supplied and price of good i respectively. Determine the equilibrium price and quantity for this two-commodity model.

Solution 3

In equilibrium, we know that the quantity supplied is equal to the quantity demanded for each good, so that

$$Q_{D_1} = Q_{S_1} \quad \text{and} \quad Q_{D_2} = Q_{S_2}$$

Let us write these respective common values as Q_1 and Q_2. The demand and supply equations for good 1 then become

$$Q_1 = 10 - 2P_1 + P_2$$

$$Q_1 = -3 + 2P_1$$

Hence

$$10 - 2P_1 + P_2 = -3 + 2P_1$$

since both sides are equal to Q_1. It makes sense to tidy this equation up a bit by collecting all of the unknowns on the left-hand side and putting the constant terms on to the right-hand side:

$$10 - 4P_1 + P_2 = -3 \quad \text{(subtract } 2P_1 \text{ from both sides)}$$

$$-4P_1 + P_2 = -13 \quad \text{(subtract 10 from both sides)}$$

Solution 3

We can perform a similar process for good 2. The demand and supply equations become

$$Q_2 = 5 + 2P_1 - 2P_2$$

$$Q_2 = -2 + 3P_2$$

because $Q_{D_2} = Q_{S_2} = Q_2$ in equilibrium. Hence

$$5 + 2P_1 - 2P_2 = -2 + 3P_2$$

$$5 + 2P_1 - 5P_2 = -2 \qquad \text{(subtract } 3P_2 \text{ from both sides)}$$

$$2P_1 - 5P_2 = -7 \qquad \text{(subtract 5 from both sides)}$$

We have therefore shown that the equilibrium prices, P_1 and P_2, satisfy the simultaneous linear equations

$$-4P_1 + P_2 = -13 \qquad (1)$$

$$2P_1 - 5P_2 = -7 \qquad (2)$$

which can be solved by elimination. Following the steps described in Section 1.2 we proceed as follows.

$$P_1 = 4 \quad P_2 = 3.$$

Exercises

1. If $f(x) = 3x + 15$ and $g(x) = {}^{1}/3x - 5$, evaluate

(a) $f(2)$ (b) $f(10)$ (c) $f(0)$ (d) $g(21)$ (e) $g(45)$ (f) $g(15)$

2. Sketch a graph of the supply function

$$P = {}^{1}/_{3}Q + 7$$

Hence, or otherwise, determine the value of

(a) P when $Q = 12$

(b) Q when $P = 10$

(c) Q when $P = 4$

6. The demand and supply functions of a good are given by

$$P = -3Q_{D} + 48$$

$$P = {}^{1}/_{2}Q_{S} + 23$$

Find the equilibrium quantity if the government imposes a fixed tax of \$4 on each good.

7. The demand and supply functions for two interdependent commodities are given by

$$Q_{D_1} = 100 - 2P_1 + P_2$$

$$Q_{D_2} = 5 + 2P_1 - 3P_2$$

$$Q_{S_1} = -10 + P_1$$

$$Q_{S_1} = -5 + 6P_2$$

where Q_{D_i}, Q_{S_i} and P_i denote the quantity demanded, quantity supplied and price of good i respectively. Determine the equilibrium price and quantity for this two-commodity model.

8.

(a) Copy and complete the following table of values for the supply function

$$P = \frac{1}{2}Q + 20$$

Q	0		50
P		25	

Hence, or otherwise, draw an accurate sketch of this function using axes with values of Q and P between 0 and 50.

(b) On the same axes draw the graph of the demand function

$$P = 50 - Q$$

and hence find the equilibrium quantity and price.

Thank you for attention!