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FACULTY OF ECONOMICS & ADMINISTRATIVE SCIENCES

MATHEMATICS FOR BUSINESS LECTURE NOTES-5

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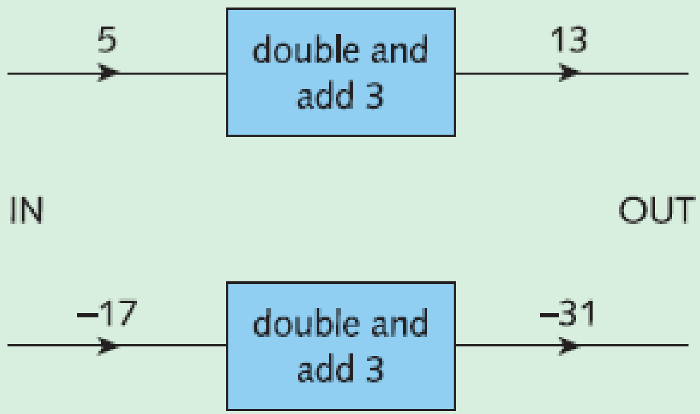
Transposition of formulae

1. Function

- A ***function,*** *f*, is a rule which assigns to each incoming number, *x*, a uniquely defined outgoing number, *y*.
- A function may be thought of as a 'black box' that performs a dedicated arithmetic calculation.
- As an example, consider the rule 'double and add 3'.
- The effect of this rule on two specific incoming numbers, 5 and −17, is illustrated in Figure 1.13.

1. Function

Figure 1.13



1. Function

- Unfortunately, such a representation is rather cumbersome.
- There are, however, two alternative ways of expressing this rule which are more concise. We can write either $y = 2x + 3$ or $f(x) = 2x + 3$.
- The first of these is familiar to you from our previous work; corresponding to any incoming number, *x*, the right-hand side tells you what to do with *x* to generate the outgoing number, *y*.

1. Function

- The second notation is also useful.
- It has the advantage that it involves the label *f*, which is used to name the rule.
- If, in a piece of economic theory, there are two or more functions, we can use different labels to refer to each one.
- For example, a second function might be $g(x) = -3x + 10$
- and we subsequently identify the respective functions simply by referring to them by name: that is, as either *f* or *g*.

1. Function

- The new notation also enables the information conveyed in Figure 1.13 to be written

$f(5) = 13$ read 'f of 5 equals 13'

$f(-17) = -31$ read 'f of –17 equals –31'

- The number inside the brackets is the incoming value, *x*, and the right-hand side is the corresponding outgoing value, *y*.

Example 1

If $f(x) = x^3 + x - 2$, find the value of

a) $f(2)$ and b) $f(-3)$

Solution 1

a)Substituting $x = 2$ into $x^3 + x - 2$ gives;

$f(2) = (2)^3 + (2) - 2$ = 8+2-2=8

b) Substituting $x = -3$ into $x^3 + x - 2$ gives;

$f(-3) = (-3)^3 + (-3) - 2$ =-27-3-2=-32

Practice 1

- Evaluate

a) $f(0)$ b) $f(-2)$ c) $f(4)$ d) $g(-1)$ e) $g(1)$ f) $g(10)$

- for the two functions

$f(x) = 2x + 10$ and $g(x) = x + 5$

- Do you notice any connection between *f* and *g* ?

1. Function

- The incoming and outgoing variables are referred to as the **independent** and **dependent** variables, respectively.
- The value of *y* clearly 'depends' on the actual value of *x* that is fed into the function.
- For example, in microeconomics the quantity demanded, *Q* , of a good depends on the market price, *P* . We might express this as: $Q = f(P)$
- Such a function is called a **demand function.**

2. Inverse Function

- The two functions, *f* and *g* , are said to be **inverse function:** that is if *f* is the inverse of *g* and, equivalently, *g* is the inverse of *f* .
- If a function *f (x)* is mapping *x* to *y*, then the inverse function of *f (x)* maps *y* to *x.*

Example 2

- Find the inverse of the following functions:

a) $f(x) = x^2$

b) $f(x) = \frac{3x+2}{4}$

c) $f(x) = x^2 - 4$

d) $f(x) = \frac{4-2x}{3}$

Solution 2

a) $f(x) = x^2$,

So, $y = x^2$ (take the square root of both side)

$\sqrt{y} = x$, so $f^{-1}(x) = \sqrt{x}$

b) $f(x) = \dfrac{3x+2}{4}$,so $y = \dfrac{3x+2}{4}$ (cross-multiplication)

$4y = 3x + 2 \rightarrow 4y - 2 = 3x \rightarrow x = \dfrac{4y-2}{3}$ $f^{-1}(x) = \dfrac{4x-2}{3}$

c) $f(x) = x^2 - 4$, so $y = x^2 - 4$

$y + 4 = x^2 \rightarrow x = \sqrt{y+4} \rightarrow f^{-1}(x) = \sqrt{x+4}$

d) $f(x) = \dfrac{4-2x}{3}$, so $y = \dfrac{4-2x}{3}$

$3y = 4 - 2x \rightarrow 2x = 4 - 3y \rightarrow x = \dfrac{4-3y}{2} \cdots \rightarrow f^{-1}(x) = \dfrac{4-3x}{2}$

Practice 2 (for students)

- Find the inverse of given functions below:

a) $f(x) = \dfrac{x+3}{4}$

b) $g(x) = \dfrac{2x+3}{5}$

c) $h(x) = -3x + 4$

3. Building a Model

- Any linear function is given in the form of:

$$f(x) = ax + b.$$

- where *a* and *b* are constant **parameters.**
- *f* is said to be a **decreasing function** if *a*<*0*(*a* is less than zero),
- *f* is said to be an **increasing function** if *a*>*0* (*a* is greater than zero).
- To be able to build a model, we need at least two points which passes through the line defined by *f* -function.

Example 3

- If a linear function *f(x)=*y gets a value of 5 when *x=5*, and value of 13 when *x=-3*, find the value of *f(-1)*?

Solution 3

- First, we should evaluate *f (x)=y .*
- To do that, let us substitute all given values as follows,

$$f(x) = ax + b \rightarrow f(5) = 5a + b = 5 \qquad \text{(I)}$$

$$f(x) = ax + b \rightarrow f(-3) = -3a + b = 13 \qquad \text{(II)}$$

- we now have two sets of equations, I and II. First eliminate *b* ,

$$5a + b = 5$$

$$\underline{3a - b = -13}$$

$8a = -8 \rightarrow a = -1$ (b ' cancelled)

by substituting a into any given equation we find $b = -10$

So, the function is $f(x) = -x + 10$. Now we can evaluate the value of $f(-1)$:

$$f(-1) = -(-1) + 10 = 11$$

Exercises

1. Make Q the subject of

$$P = 2Q + 8$$

Hence find the value of Q when $P = 52$.

4. Make x the subject of each of the following formulae:

(a) $y = 9x - 6$ (b) $y = (x + 4)/3$ (c) $y = \frac{x}{2}$

(d) $y = \frac{x}{5} + 8$ (e) $y = \frac{1}{x+2}$ (f) $y = \frac{4}{3x-7}$

5. Transpose the formulae:

(a) $Q = aP + b$ to express P in terms of Q

(b) $Y = aY + b + I$ to express Y in terms of I

6. Make x the subject of the formula

$$y = \frac{3}{x} - 2$$

7.

- If a linear function $f(x)$=y gets a value of -4 when $x=2$, and value of 10 when $x=1$, find the value of $f(1)$?

8.

Make x the subject of the formula

a) $\frac{1}{7}x - 2 = y$

b) $y = \sqrt{\frac{x}{5}}$

c) $y = \frac{4}{2x + 1}$

d) $y = \frac{x^2 - 4}{2} + 1$

Thank you for attention!