Mathematics for Business: Lecture Notes -2

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# 2. Further Algebra

We are going to cover further algebra in three manageable subsections:

*Fractions*, *equations*, and *inequalities.*

## 2.1 Fractions

For a numerical fraction such as , the number 5- on the top- is called the **numerator** and the number 8- on the bottom- is called **denominator**. We will also be seeing fractions where numerator and/or denominator involve letters as well as numbers. For example, , ,  and so on. These are also algebraic fractions.

Two fractions are said to be equivalent if they represent the same numerical value. We know that is equivalent to since they are both equal to the decimal numbers 0.5. Formally we say that when the numerator and denominator are both multiplied by the same number the value of the fraction remains unchanged. It this example we have :



This process can be reversed so equivalent fractions are produced when the numerator and denominator are both divided by the same number. For example:



so the fraction  and are equivalent.

A fraction is said to be in its simplest form reduced to its lowest terms when there are no factors common to both the numerator and denominator. To express any given fraction in its simplest form you need to find the highest common factor of the numerator and denominator and then divide the top and bottom of the fraction by this.

**Example 1**. Reduce each of the following fractions to its lowest terms:

a)= b) c) d) e)

f) g) h) i) j)

**Solution 1:**

a) =(the highest common factor of 6 and 36 is 6)

b) (the highest common factor of 14 and 56 is 14)

c) ( the highest common factor of 22 and 77 is 11)

d)  (the highest common factor of 15 and 55 is 5)

e) ( the highest common factor of 39 and 65 is 13)

f) (the factor *x* is common to both *x2* and *3x* so we divide top and bottom by *x*, that

is, we cancel the *x*’s)

g)  (factorising denominator gives 3(*a*+*b*), the factor (*a*+*b*) is common to the

nominator and denominator so we cancel the (*a*+*b*)’s)

h) (factorise denominator, is common factor to both nominator and

denominator, so they cancel out)

i)  (factorise denominator, the factor is common to both nominator

and denominator, so they cancel out)

j) (factorise denominator, the factor is common to both nominator

and denominator, so they cancel out)

The rules for multiplication and division are as follows:

To **multiply** fractions you multiply their corresponding numerators and denominators:



To **divide** by a fraction you turn it upside down and multiply:



**Example 2**: Calculate:

a)= b)= c)= d)=

**Solution 2**:

a) 

b) 

c) 

d) 

The rules for addition and subtraction are as follows:

To add or subtract two fractions you write them as equivalent fractions with a common denominator and add or subtract theirs numerators.

**Example 3**: Calculate:

a) b) c)

**Solution 3**:

a) (the fractions have denominator 4 and 2 respectively. One number that is

divisible by both 4 and 2 is 4, so we choose this as the common denominator)

b) 

c) 

**Practice 1**. Find expressions for each of the following:

a) b) c) d) e)

f) g) h)

## 2.2 Equations

We have seen how to write expressions in s simpler but equivalent form. For example when we write things line,

 (collecting like terms)

or

 (writing them as equivalent fractions with a common denominator)

Please note that the left-hand and right hand side of the equations are equal (as you have this knowledge at the back of your mind). Each statement is true for all possible values of . For this reason the above relationships are called **identities**. Compare these with statements such as:

, , or and so on.

These relationships are called equations and are only true for particular values of which need to be found.

Can we solve these equations for  ? Yes, we can solve these equations using the rules of mathematics. In fact, the only rule that we need is:

“Apply whatever mathematical operation you like to an equation, provided that you do the same thing to both sides”

For example, if we are to solve the equation  for , we apply the followings in order:



 (subtract 1 from both sides)

 (divide both sides by 4)



**Example 1**: Solve

a) b) c)

d) e)

**Solution**:

a) 

 (subtract 6x from both sides)

 (add 9 to the both sides)

 (divide both side by 4)



b) 

 (multiply out the brackets)

 (collect like terms)

 (add 1 to the both sides)

 (divide both side by 7)

c) 

 (multiply both sides by )

 (’s cancels out)

 (multiply out the brackets)

 (add 7 to the both sides)

 (simplify)

d) 

 (cross-multiplication: if , then )

 (multiply out the brackets)

 (subtract  from both sides)

 (subtract 9 from both sides)

 (divide both side by 11)

e) 

 (square both sides of the equation)

 (multiply both sides by )

 (multiply out the brackets)

 (subtract  from both sides)

 (divide both sides by -2)