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MATHEMATICS FOR BUSINESS LECTURE NOTES-11

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Simple Interest

- Today, businesses and individuals are faced with a bewildering array of loan facilities and investment opportunities.
- In this section we explain how these financial calculations are carried out to enable an informed choice to be made between the various possibilities available.
- We begin by considering what happens when a single lump sum is invested and show how to calculate the amount accumulated over a period of time.

- Suppose that someone gives you the option of receiving $500 now or $500 in 3 years' time.
- Which of these alternatives would you accept?
- Most people would take the money now, partly because they may have an immediate need for it, but also because they recognize that $500 is worth more today than in 3 years' time.
- Even if we ignore the effects of inflation, it is still better to take the money now, since it can be invested and will increase in value over the 3-year period.

- Simple interest is only used on short-term notes- often of duration less than 1 year.
- The concept of simple interest, however, forms the basis of much of the rest of the material developed in this lecture, for which time periods may be much longer than a year.
- If you deposit a sum of money *P* in a savings account or if you borrow a sum of money *P* from a lending agent, then *P* is referred to as the principal.
- When money is borrowed -whether it is a savings institution borrowing from you when you deposit money in your account or you borrowing from a lending agent- a fee is charged for the money borrowed.
- This fee is rent paid for the use of another's money, just as rent is paid for the use of another's house.
- The fee is called interest. It is usually computed as a percentage (called the interest rate) of the principal over a given period of time.
- The interest rate, unless otherwise stated, is an annual rate.

- Simple interest is given by the following formula:

Simple interest ; $I = P \times r \times t$

where

P : principal

r : annual sample interest rate (written as a decimal)

t :time in years

Example 1.

- Calculate an interest on a loan of $500 invested for 1 year at 10% interest compounded annually.

Solution 1

- *I=P* x *r* x *t*
- *I=500* x *0.10* x *1=50*
- At the end of 1 year, the borrower would repay the principal ($500) plus the interest ($50), or a total of $550.

Amount : Simple Interest

$$A = P + P \times r \times t = P((1 + r \times t)$$

where

P : principal

r : annual sample interest rate (written as a decimal)

t :time in years

A : amount, or future value

Example 2

- Calculate the total amount the borrower would repay on a loan of $1000 invested for 1 year at 8% interest compounded annually.

Solution 2

- $A=P((1 + r \times t))$
- $A=1000 \times (1 + 0.08 \times 1)= 1080$
- At the end of 1 year, the borrower would repay the principal ($1000) plus the interest ($80), or a total of $1080.

Compound Interest

- If at the end of a payment period the interest due is reinvested at the same rate, then the interest as well as the original principal will earn interest during the next payment period.
- Interest paid on interest reinvested is called compound interest.

Example 3

- Suppose you deposit $500 in a bank that pays 10% compounded annually. How much will the bank owe you at the end of 3 years?

Solution 3

- What exactly do we mean by '10% interest compounded annually'?
- Well, at the end of each year, the interest is calculated and is added on to the amount currently invested.
- If the original amount is $500 then after 1 year the interest is 10% of $500, which is

$$A = P(1 + rt)$$
$$A = 500(1 + 0.10 \times 1)$$
$$A = 500(1.1) = \$550$$

Solution 3

- Under annual compounding the interest obtained at the end of the second year is 10% of the amount invested at the start of that year. This not only consists of the original $500, but also the $50 already received as interest on the first year's investment.
- Consequently, we get an additional

$$A = P(1 + rt)$$
$$A = 550(1 + 0.10 \times 1)$$
$$A = 550(1.1) = \$605$$

- raising the sum to $605. Finally, at the end of the third year, the interest is

$$A = P(1 + rt)$$
$$A = 605(1 + 0.10 \times 1)$$
$$A = 605(1.1) = \$665.5$$

Solution 3

- Let us look over the calculations:

$A = 500(1.10)$ *end of first year*

$A = 500(1.10)(1.10) = 500(1.10)^2$ *end of second year*

$A = 500(1.10)(1.10)(1.10) = 500(1.10)^3$ *end of third year*

- It appears that at the end of *n* years, we would have:

$A = 500(1.10)^n$ *end of $n-th$ year*

- To summarize, compounded interest is calculated based on the general formula below:

$$A = P(1+i)^n$$

Where $i = \dfrac{r}{m}$ and

r :annual interest rate

m :number of compounding periods per year

i :rate per compounding period

n :total number of compounding periods.

P :principal(or present value)

A : amount (future value) at the end of n periods.

- The compound interest formula derived above involves four variables, r, n, P and S.
- Provided that we know any three of these, we can use the formula to determine the remaining variable. This is illustrated in the following example.

Example 4

- A principal of $25 000 is invested at 12% interest compounded annually. After how many years will the investment first exceed $250 000?

Solution 4

$$S = P\left(1 + \frac{r}{100}\right)^n$$

We are given that

$$P = 25\,000,\ S = 250\,000,\ r = 12$$

so we need to solve the equation

$$250\,000 = 25\,000\left(1 + \frac{12}{100}\right)^n$$

$$10 = (1.12)^n$$

$$\log(10) = \log(1.12)^n$$

$$\log(10) = n\ \log(1.12)$$

$$n = \frac{\log(10)}{\log(1.12)}$$

$$= \frac{1}{0.049\,218\,023}$$

$$= 20.3$$

- You may have noticed that in all of the previous problems it is assumed that the interest is compounded annually.
- It is possible for interest to be added to the investment more frequently than this.
- For example, suppose that a principal of $500 is invested for 3 years at 10% interest compounded quarterly. What do we mean by '10% interest compounded quarterly'?
- Well, it does not mean that we get 10% interest every 3 months. Instead, the 10% is split into four equal portions, one for each quarter. Every 3 months the interest accrued is

$$\frac{10\%}{4} = 2.5\%$$

so after the first quarter the investment gets multiplied by 1.025 to give

$$500(1.025)$$

and after the second quarter it gets multiplied by another 1.025 to give

$$500(1.025)^2$$

and so on.

Example 5

- Find the total amount due on a loan of \$800 at 9% annual interest at the end of 4 months.

Practice (for students)

- **Practice 1:** If you want to earn an annual rate of 10% on your investments, how much (to the nearest cent) should you pay for a note that will be worth 5000 in 9 months?

- **Practice 2:** T-bills (treasury bills) are one of the instruments the U.S. Treasury Department uses to finance the public debt. If you buy a 180-day T-bill with a maturity value of $10,000 for $9,693.78, what annual interest rate will you earn? (express the answer as a percentage, up to 2 decimal places)

- **Practice 3:** A loan of $7250 was repaid at the end of 8 months. What size repayment checks (principal and interest) was written, if a 9% annual rate of interest was charged?

- **Practice 4:** If you paid $120 to a loan company for the use of $2000 for 90 days, what annual rate of interest did they charge?

Example 6

- If $1000 is invested at 8% compounded

a)annually

b)semi-annually

c)quarterly

d)monthly

- what is the amount after 5 years?

Solution 6

a)Compounding annually means there is one interest payment period per year. Thus, n =5, $i = r = 0.08$

$$A = P(1+i)^n$$
$$= 1000(1+0.08)^5$$
$$1000(1,469328)$$
$$= 1469,33$$

b) Compounding semi-annually means there are two interest payment periods per year. Thus, the number of payment periods in 5 years is n =2x5=10, and the interest rate per period is

$$i = \frac{r}{m} = \frac{0.08}{2} = 0.04$$

$$A = P(1+i)^n$$
So, $= 1000(1+0.04)^{10}$
$$= 1480.24$$

Solution 6

c) Compounding quarterly means there are four interest payment periods per year. Thus, the number of payment periods in 5 years is n =4x5=20, and the interest rate per period is

$$i = \frac{r}{m} = \frac{0.08}{4} = 0.02 \text{ so,}$$

$$A = P(1+i)^n$$

$$= 1000(1+0.02)^{20}$$

$$= 1485.95$$

d) Compounding monthly means there are twelve interest payment periods per year. Thus, the number of payment periods in 5 years is n =12x5=60, and the interest rate per period is

$$i = \frac{r}{m} = \frac{0.08}{12} = 0.000666 \text{ so,}$$

$$A = P(1+i)^n$$

$$= 1000(1+0.000666)^{60}$$

$$= 1489.85$$

Practice

- **Practice 5:** How much should you invest now at 10% compounded quarterly to have 8000 toward the purchase of a car in 5 years?

- **Practice 6:** A new born child receives a $5000 gift toward a college education from her grandparents. How much will the $5000 be worth in 17 years if it is invested at 7% compounded quarterly?

- **Practice 8:** If an investment company pays 8% compounded semi-annually, how much should you deposit now to have $6000

  a) 3 years from now? b) 6 years from now?

Exercises

1. A bank offers a return of 7% interest compounded annually. Find the future value of a principal of $4500 after 6 years.

2. Find the future value of $20 000 in 2 years' time if compounded quarterly at 8% interest.

3. The value of an asset, currently priced at $100 000, is expected to increase by 20% a year.

   (a) Find its value in 10 years' time.

   (b) After how many years will it be worth $1 million?

Thank you for attention!